Sample problems with solutions

1. A class consists of boys, girls, and Martians in a ratio of 20: 28: 2. There are 168 girls. What is the class total?

Answer: Let x = the proportionality factor. 28x = 168, divide both sides by 28, so x = 6. Total = 20x + 28x + 2x = 20*6 + 28*6 + 2*6 = 300.

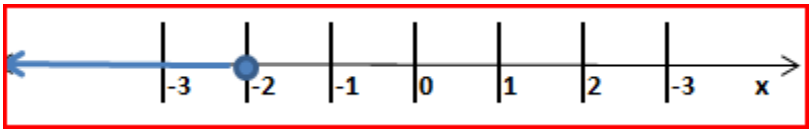
2. Place these numbers in order from smallest to largest; round final answers to 3 decimal places:

$-\pi/2$, $-\sqrt{(\pi/2)}$, $-\sqrt{(\pi)}/2$, $-(\sqrt{(\pi/2)})^{1/2}$, $-(\sqrt{(\pi/2)})^{-1/2}$

Answer: Use the calculator which has a square root key but not a π key, and approximate π as 3.14. $-\pi/2 \approx -1.570$, $-\sqrt{(\pi/2)} \approx -1.253$, $-\sqrt{(\pi)}/2 \approx -.886$, $-(\sqrt{(\pi/2)})^{1/2} \approx -1.119$, $-(\sqrt{(\pi/2)})^{-1/2} \approx -.893$. Answer: $-\pi/2$, $-\sqrt{(\pi/2)}$, $-(\sqrt{(\pi/2)})^{1/2}$, $-(\sqrt{(\pi/2)})^{-1/2}$, $-\sqrt{(\pi)}/2$.

3. Solve for x: $-3x + 7 \geq 13$. Plot the solution on a number line.

Answer: $-3x + 7 \geq 13$, subtract 7 from both sides to get $-3x \geq 6$, divide by -3 and change the direction of the inequality, $x \leq -2$. The number line looks as follows. Note that at -2 there is a filled-in circle indicating -2 is part of the solution. If the solution had been $x < -2$, then the circle would not be filled in.



4. Suppose I drive for three-and-a-half hours, of which some is at 70 mph and some is at 30 mph. If the total distance is 225 miles, how much time was spent at each speed?

Answer: Let t = time at 70 mph, and 3.5-t = time at 30 mph. Then 70t + 30(3.5-t) = 225. 70t + 105 - 30t = 225, 40t = 120, t = 3, 3.5-t=.5. (Be careful that units are consistent, such as time in hours and speed in mph.)

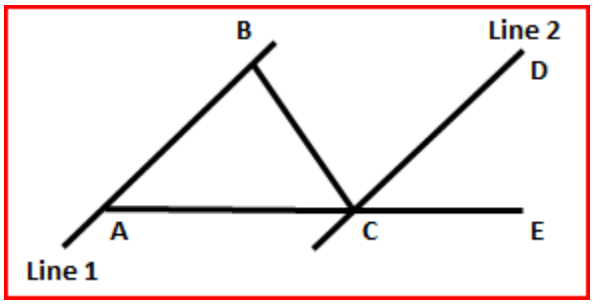
5. A two-digit number has the property that the ten's digit is twice the unit's digit. If the number is reversed, the sum of the original number and the reversed number is 132. What is the original number?

Answer: In a two-digit number like 37, 3 is the ten's digit, 7 is the unit's digit, and 37 equals (10*3) + 7. Let u = the unit's digit of the original number, and 2u = ten's digit of original number. The original number equals (10*2u) + u, and the reversed number equals (10*u) + 2u. Then (10*2u) + u + (10*u) + 2u = 132. 20u + u + 10u + 2u = 33u = 132. u=4, the original number is 84.

6. A movie theatre sold 200 tickets. Some customers paid the regular price of $8 per ticket, and some customers had a coupon entitling them to a 25% discount. The total sales were $1460. How many of each type of ticket were sold?

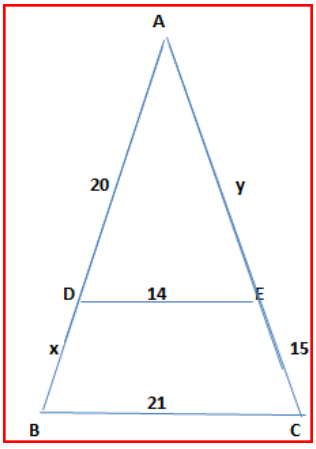
Answer: A 25% discount on an $8 ticket is a discount of .25*8=2, so the discounted ticket is 8-2=6. Let x = number of $8 tickets, and 200-x = number of $6 tickets. 8x + 6(200-x) = 1460. 8x + 1200 – 6x = 1480, 2x = 260, x = 130, 200-x = 70. So there were 130 $8 tickets and 70 $6 tickets.

7. Let lines 1 and 2 be parallel, ∠CAB = 60°, and ∠ABC = 70°. Calculate the measure of ∠BCD and ∠DCE.



Answer: ∠ABC and ∠BCD are a pair of alternate interior angles, so ∠BCD = 70°. ∠DCE = 180°-50°-70° = 60°.

8. In the diagram below, line segments DE and BC are parallel, AD=20, DB=x, AE=y, EC=15, DE=14, and BC=21. Find x and y.



Answer: Because of the parallel lines, corresponding angles are equal, so Δ ADE ~ Δ ABC. Therefore, the lengths of corresponding sides are proportional, in the ratio 14:21, or 2:3. Then 20/(20+x) = 2/3 and y/(y+15) = 2/3. The first equation gives x=10, and the second equation gives y=30.

9. What is the measure of the central angle of a circular clock formed by the hands of the clock at 4:30?

Answer: There are 12 numbers on a clock, and 360 degrees to a circle, so each number represents 360/12 = 30 degrees. At 4:30, the small hand is on 4.5 and the big hand is on 6, a difference of 1.5. So this 1.5 represents 1.5 * 30 degrees = 45 degrees.

10. Use the following table to calculate the probability that if a person is chosen at random, that person will be Female, given that the person is currently married.

	Currently unmarried	Currently married	Total
Male	25	65	90
Female	30	80	110
Total	55	145	200

Answer: There are 145 people currently married, and among them 80 are female. The answer is 80/145.

END

Jerome Tuttle